

N88-11172

## CONSTITUTIVE MODELING FOR ISOTROPIC MATERIALS\*

V.G. Ramaswamy, R.H. Van Stone, L.T. Dame, J.H. Laflen  
General Electric Company  
Aircraft Engine Business Group

## INTRODUCTION

Accurate analysis of stress-strain behavior is of critical importance in the evaluation of life capabilities of hot section turbine engine components such as turbine blades and vanes. The constitutive equations used in the finite element analysis of such components must be capable of modeling a variety of complex behavior exhibited at high temperatures by cast superalloys. The classical separation of plasticity and creep employed in most of the finite element codes in use today is known to be deficient in modeling elevated temperature time dependent phenomena. Rate dependent, unified constitutive theories can overcome many of these difficulties and are more suitable for the analysis of the complex behavior of high temperature superalloys. However, many aspects of the unified theories had not been fully evaluated prior to the initiation of this work, and thus these theories had not been generally accepted for use in engine hardware design.

In this contract, constitutive theories were evaluated against a large uniaxial and multiaxial data base that was generated as a part of the work. Initially it was the intent to evaluate only available theories, however it was found that no available approach was satisfactory in modeling the high temperature time dependent behavior of Rene' 80 which is a cast turbine blade and vane nickel base superalloy. Additional considerations in model development included the cyclic softening behavior of Rene' 80, rate independence at lower temperatures, and the development of a new model for static recovery. The final constitutive model was implemented into a finite element computer code which was developed as a part of the contract and which was developed specifically for use with unified theories. The code was verified by a re-analysis of the Turbine Tip Durability problem which was part of the pre-HOST activities at General Electric [1]. This paper summarizes the work on the second year of the contract; the first year's work was summarized in last year's HOST Conference [2] and in the Annual Report [3]. Details of the contractual efforts are given in the final report which is currently being reviewed.

## EXPERIMENTAL PROGRAM

The experimental approach in this work was to determine the constitutive behavior of Rene' 80 under a multitude of conditions that are important in the design of gas turbine blade and vanes. This required that thin wall specimens be used as other NASA sponsored work has shown that thickness can be an important factor in material response properties [4]. Consequently, all

---

\*Work done under NASA Contract NAS3-23927.

specimens were designed with a thickness of 0.03 inches. Similarly, the experimental temperatures, strain ranges, strain ratios, hold times, and strain rates were established through an evaluation of the operating conditions in commercial jet engines.

In performing the experiments, the approach was to evaluate a series of transient and steady state conditions in each specimen test by using a block cycling method. This is illustrated in Figure 1 where several blocks of strain ranges are used in one experiment - all other conditions, strain rate, temperature, etc. are kept constant. By using several combinations of strain range blocks in different sequences in a single test all combinations of transient effects could be interrogated. The block length was selected to produce cyclically stable hysteresis loops by the end of each block.

All uniaxial experiments were performed at the EMTL Laboratory of the General Electric Company in Evendale, Ohio. The experimental results of each test were automatically saved in digitized form in real time by using a ETS data acquisition device. Prior to test, the number of load-strain pairs to be saved in selected hysteresis loops was determined (between 200 to 400 points). Typically 50 hysteresis loops were saved in each test. After a test, the data were permanently saved on tapes, before being loaded into mainframe computer files which were used for detailed data analyses. Final data analysis would include determining time derivatives by fitting a second order polynomial to seven consecutive data points and differentiating. These final data files could then be used in plotting data or determining constants in constitutive theories.

Similarly, the tension torsion test results were available on cassette tapes which could be used for detailed data analyses. These tests were performed at the Turbine Technology Laboratory (TTL) of the General Electric Company in Schenectady, New York under the direction of Dr. R. Williams. These tests used a development extensometer from the Instron Corporation. Additionally, a series of notched specimen test were conducted at Michigan State University under the direction of Professor J. Martin. In these tests, the notch root strains were measured using an interferometric displacement gage.

## THEORY DEVELOPMENT

Following a detailed literature review, General Electric selected the Bodner model and a generic drag stress/back stress model for further detailed evaluations with the Rene' 80 data. The intent in the evaluations with the generic model was to perform an analysis of the Rene' 80 data to determine which functional forms in suggested back stress/drag stress models were most appropriate. Many of the results were presented in last year's paper. It was found that neither the Bodner or the generic model were very adequate for predicting the response properties of Rene' 80 at 1800°F. Consequently a new theory was developed which combined the Bodner exponential flow law with a

back stress formulation. The Bodner flow law had been found to be superior in modeling the strain rate behavior of Rene' 80, and possess the ability to predict essentially rate independent behavior at lower temperatures. These are two essential aspects of the behavior of Rene' 80. Additionally, it was found to be necessary to modify the evolution equations for the back stress to account for static recovery effects and to account for effects in the small inelastic strain regime. When these factors were included the final set of equations could be written as:

$$\dot{\epsilon}_{ij}^I = D \exp \left[ -\frac{A}{2} \left( \frac{Z^2}{3K_2} \right)^n \right] \frac{(S_{ij} - \Omega_{ij})}{\sqrt{K_2}} \quad (1)$$

$$K_2 = 1/2 (S_{ij} - \Omega_{ij}) (S_{ij} - \Omega_{ij}) \quad (2)$$

$$\Omega_{ij} = \frac{G}{E} S_{ij} + (1 - \frac{G}{E}) \Omega_{ij}^I \quad (3)$$

$$\Omega_{ij}^I = f_1 \dot{\epsilon}_{ij}^I - \frac{f_1}{\Omega_s} \Omega_{ij} \dot{R} \quad (4)$$

$$\dot{Z} = m(Z_1 - Z) \dot{W}^I \quad (5)$$

$$\Omega_s = -B(\sigma_e/\sigma_0)^r (\Omega_s - \Omega_{sat}) \quad (6)$$

where

$$\dot{R} = \sqrt{\frac{2}{3} \dot{\epsilon}_{ij}^I \dot{\epsilon}_{ij}^I},$$

$$\Omega_s(0) = \Omega_{max}, \Omega_{ij}^I(0) = 0, z(0) = Z_0$$

The procedures for determining the constants in these equations have been presented previously in [2,3]. Basically the procedure involves the determination of most of the constants through the use of the monotonic strain rate dependent stress-strain curves; only the saturated value of the drag stress,  $Z_1$ , is determined from cyclic tests (saturated hysteresis loops from fully reversed cyclic tests are used in this case). Verification of this method of determining the constants is presented in the form of correlations with multiaxial, hold time/relaxation, and different mean strain test data.

That these equations are very accurate is shown through comparisons with data. Figures 2, 3 and 4 show the correlation of the monotonic stress-strain data at 1400°F through 1800°F, respectively. That the theory is capable of predicting strain rate dependent as well as rate independent behavior is

apparent. Figure 5 shows a prediction of a compressive mean strain test result. Figure 5(a) shows a comparison with the first two cycles, while Figure 5(b) shows a comparison with the saturated hysteresis loop of this test. Figures 6 and 7 show that the theory can predict the stress relaxation behavior at high and low temperatures. Note that while the monotonic data at 1400°F exhibits little rate dependence there is still stress relaxation. Creep comparisons are shown in Figures 8 and 9. These predictions depend strongly on the form of the static recovery term. Note that the form that is used in the current theory is much different than those used in other unified approaches.

Multiaxial comparisons are shown in Figures 10 to 13. Figures 10 (a) and (b) show the axial and torsion comparisons, respectively, for a combined tension-torsion (in-phase) test. Figure 11 shows the data results and theory predictions from a special nonproportional test where segments of proportional cycles were used (see the insert of these two figures). Figure 11 compares results from the first segment (cycle 5), and the last segment (cycle 32) of this nonproportional loading experiment. That the predictions are accurate illustrates that the theory is good for such conditions without considering the additional hardening that has been found in some other materials. Figures 12 and 13 show that the new theory can predict 90 deg. out of phase tension/torsion experimental results at two temperatures with good accuracy.

Figures 14 and 15 show two predictions of the theory with test data from combined temperature and strain cycling tests. The predictions are shown to be reasonable considering that the predictions are based only on isothermal test data.

#### FINITE ELEMENT IMPLEMENTATION

The theory discussed in the previous section was implemented into a new 3-D finite element code which uses a 20-noded brick element. The program uses a dynamic time incrementing procedure to minimize cost while guaranteeing an accurate solution. The inelastic rate equations and state variable evolution equations are integrated using a second order Adams-Moulton predictor corrector technique. Piecewise linear load histories are modeled in order to simplify input. Further economics have been achieved by improving the stability of the initial strain method and further reducing the number of equilibrium iterations. The program was developed on an IBM PC AT and has been successfully installed on the NASA-Lewis CRAY computer.

#### SUMMARY

The experimental and analytical goals of this program were successfully accomplished. A new multiaxial constitutive model which can represent the complex nonlinear high temperature behavior of Rene' 80 was developed. The

model was extensively verified based on data at several temperatures. The TMF and nonproportional cyclic modeling capabilities of the model were demonstrated. The model was implemented in a 3-D finite element code which was installed on the NASA-Lewis CRAY.

#### REFERENCES

1. McKnight, R.L., Laflen, J.H., and Spamer, G.T. "Turbine Blade Tip Durability Analysis", NASA CR165268, February 1982.
2. Ramaswamy, V.G., Van Stone, R.H., Dame, L.T., and Laflen, J.H., "Constitutive Modeling for Isotropic Materials," NASA Conference Publication 2339, October 1984.
3. Ramaswamy, V.G., Van Stone, R.H., Dame, L.T., and Laflen, J.H., "Constitutive Modeling for Isotropic Materials", Annual Report, NASA CR17485, March 1985.

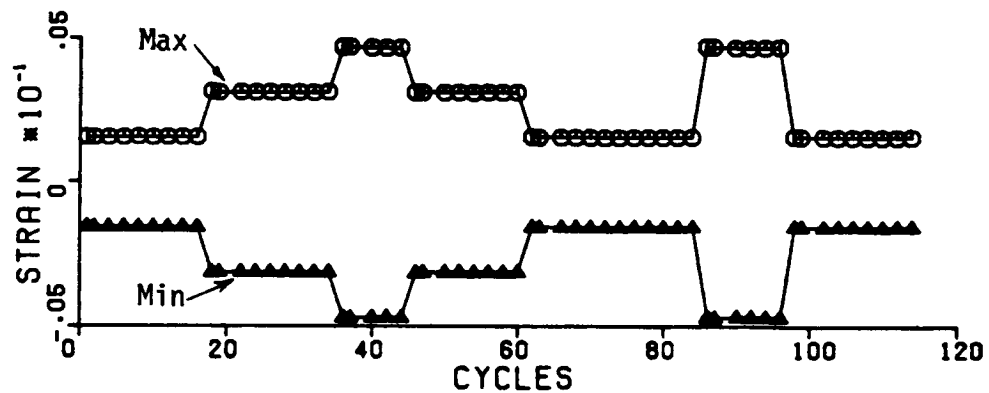


Figure 1. Typical Strain Limits Imposed During Cyclic Experiments

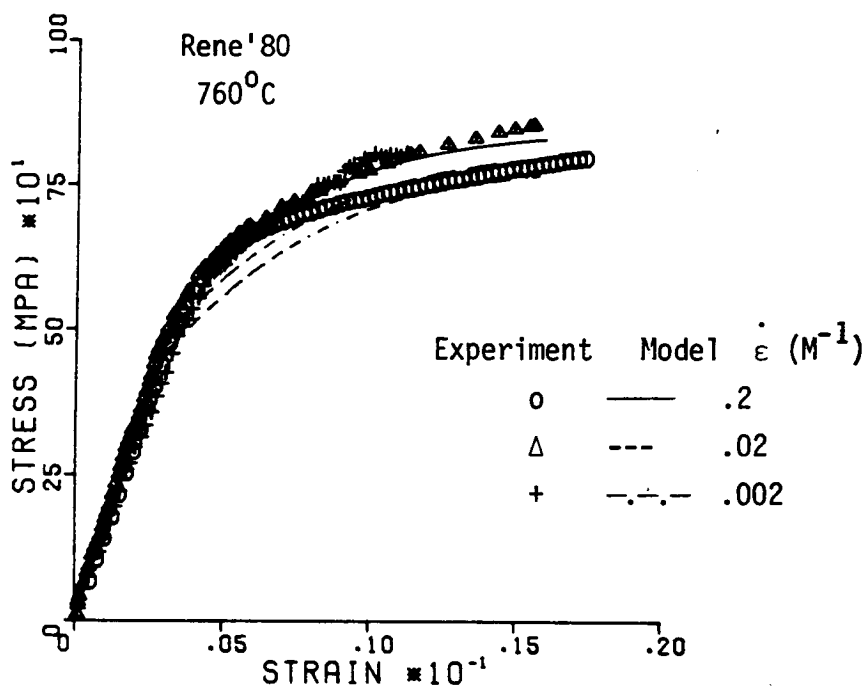


Figure 2. Monotonic Tensile Response of Rene'80 at 760°C

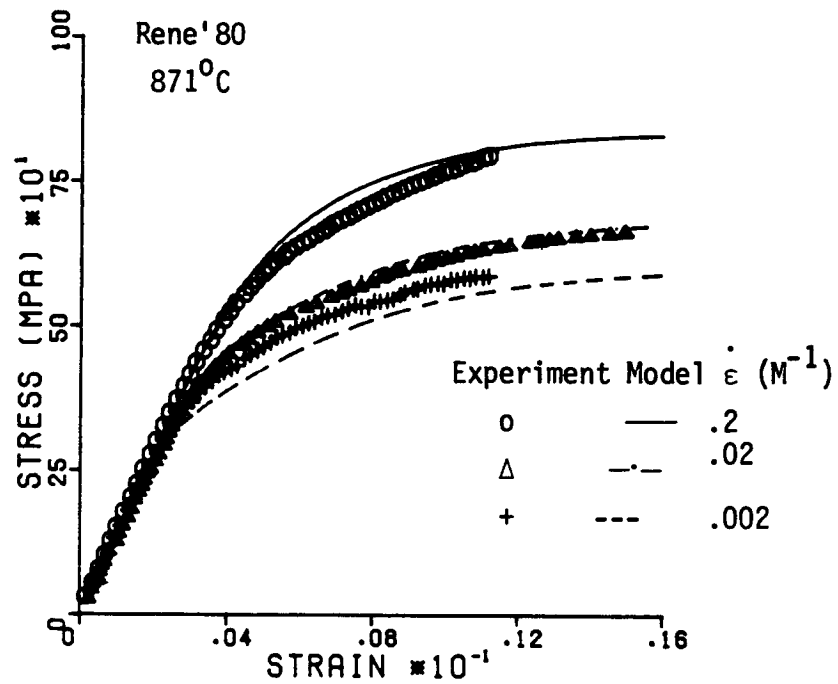


Figure 3. Monotonic Tensile Response of Rene'80 at 871°C

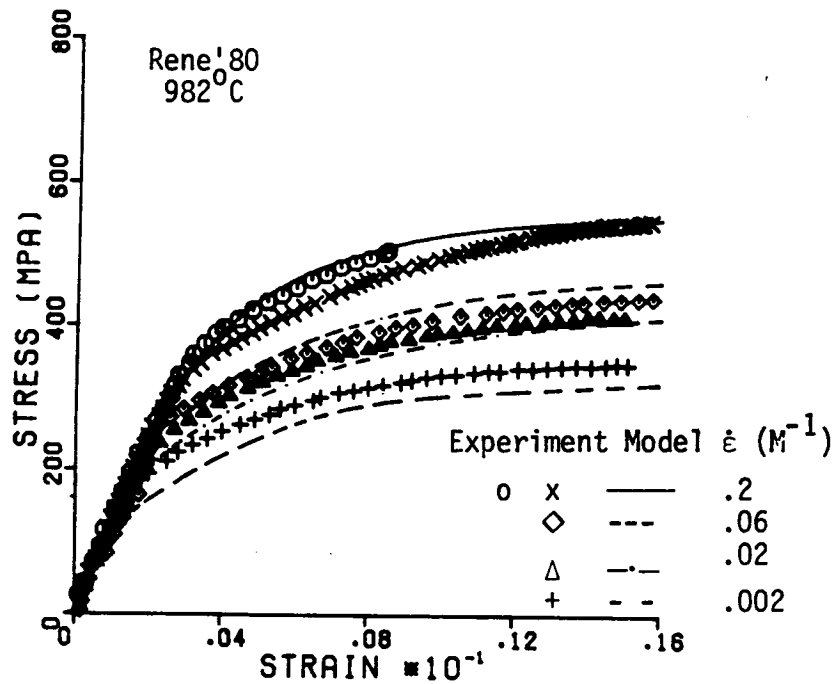
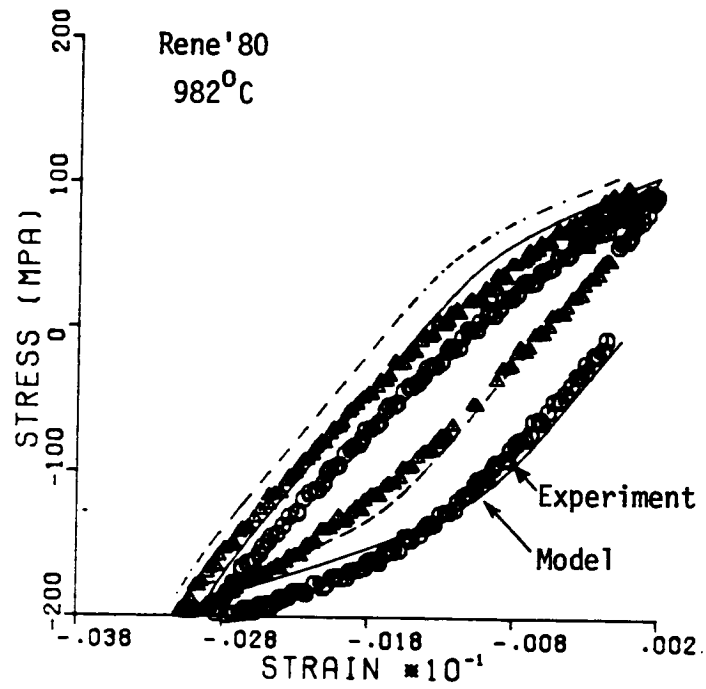
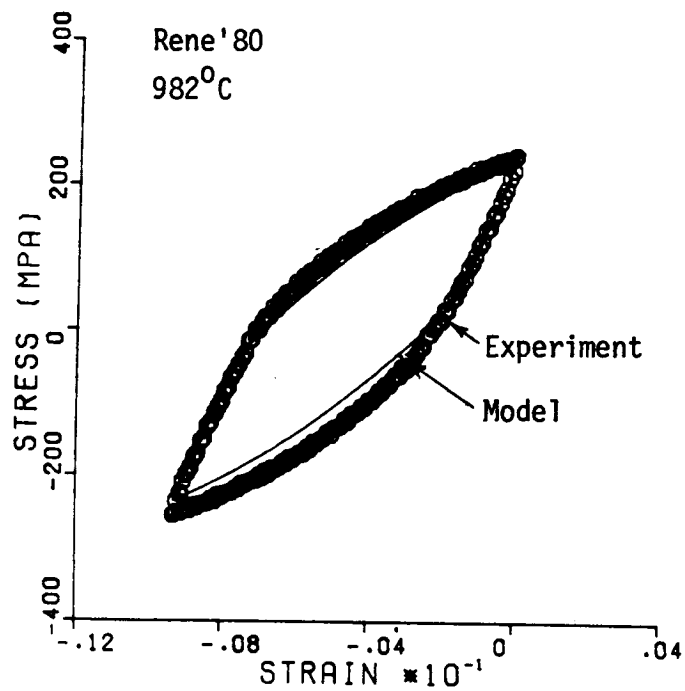


Figure 4. Monotonic Tensile Response of Rene'80 at 982°C



(a) First Two Cycles



(b) Cycle 78

Figure 5. Cyclic Response of Rene'80 with Mean Strain



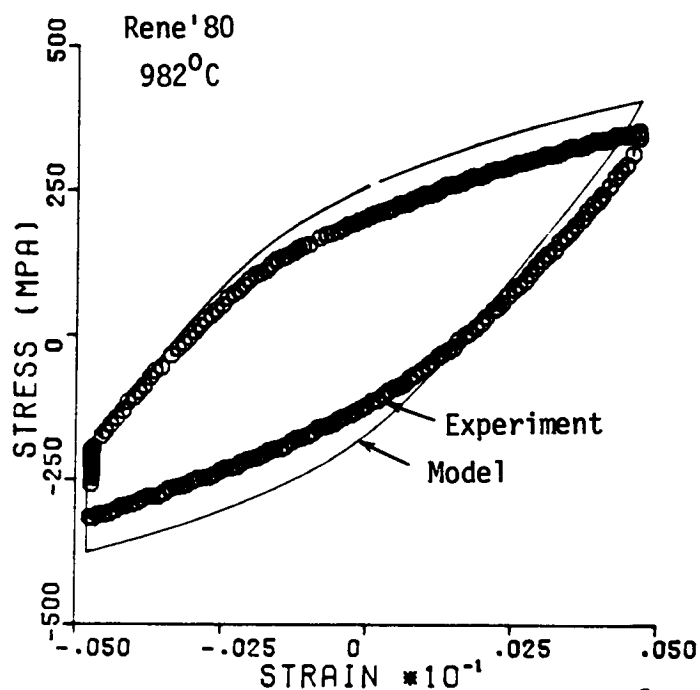


Figure 6. Stress Relaxation in Rene'80 at 982°C  
(Cycle No. 60)

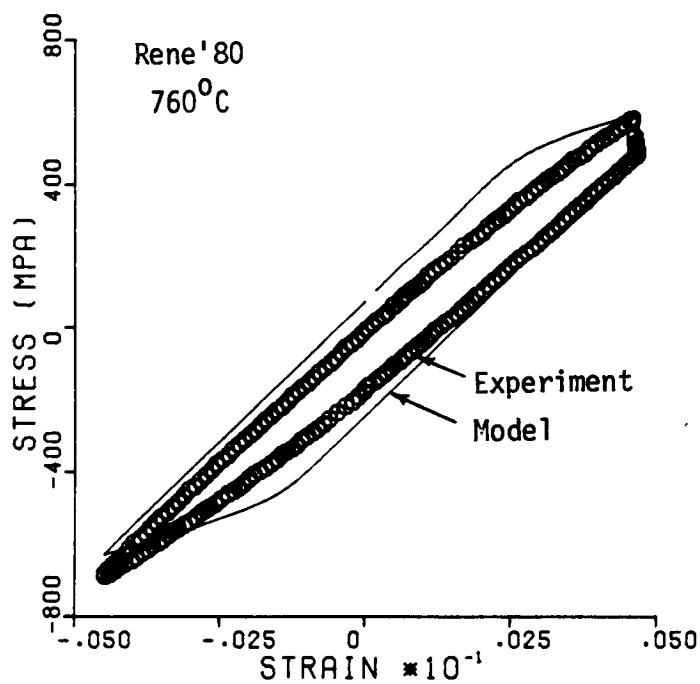


Figure 7. Stress Relaxation in Rene'80 at 760°C  
(Cycle No. 94)

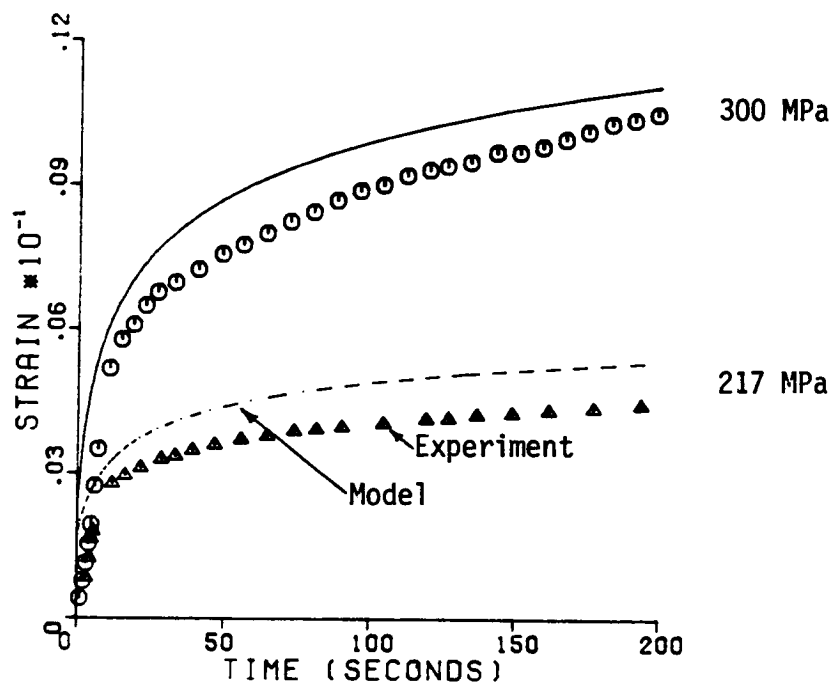


Figure 8 Creep Response of Rene'80 at 982°C

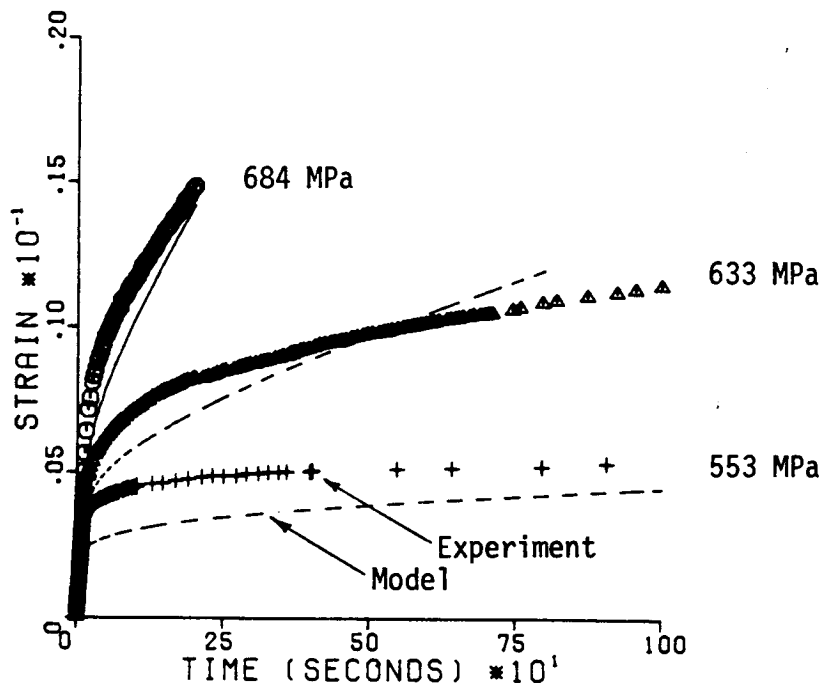
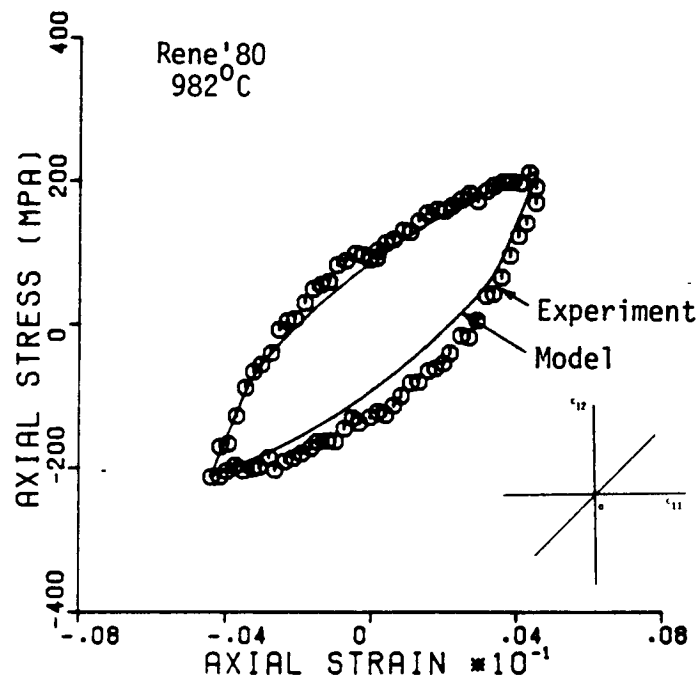
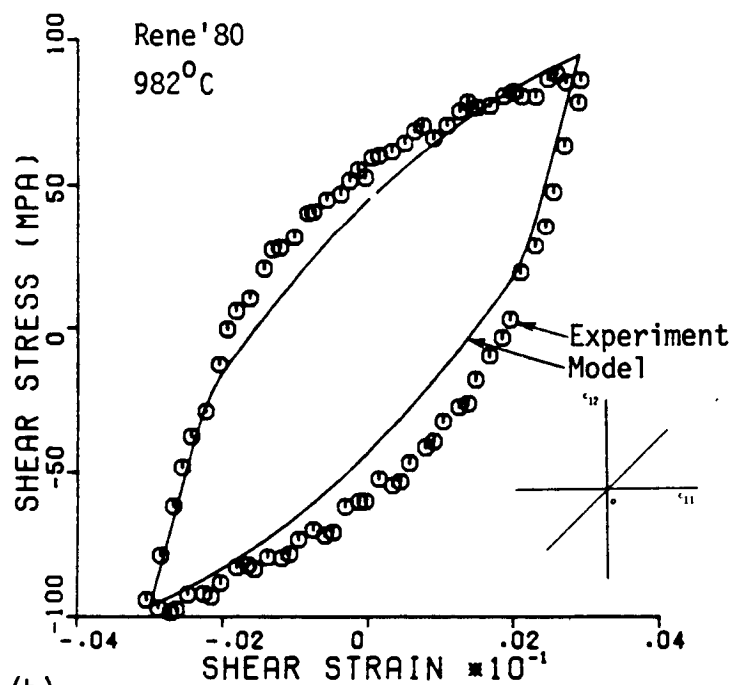


Figure 9 Creep Response of Rene'80 at 760°C



(a)



(b)

Figure 10. In phase Tension Torsion Cyclic Response of Rene'80 at 982°C,  $.002M^{-1}$

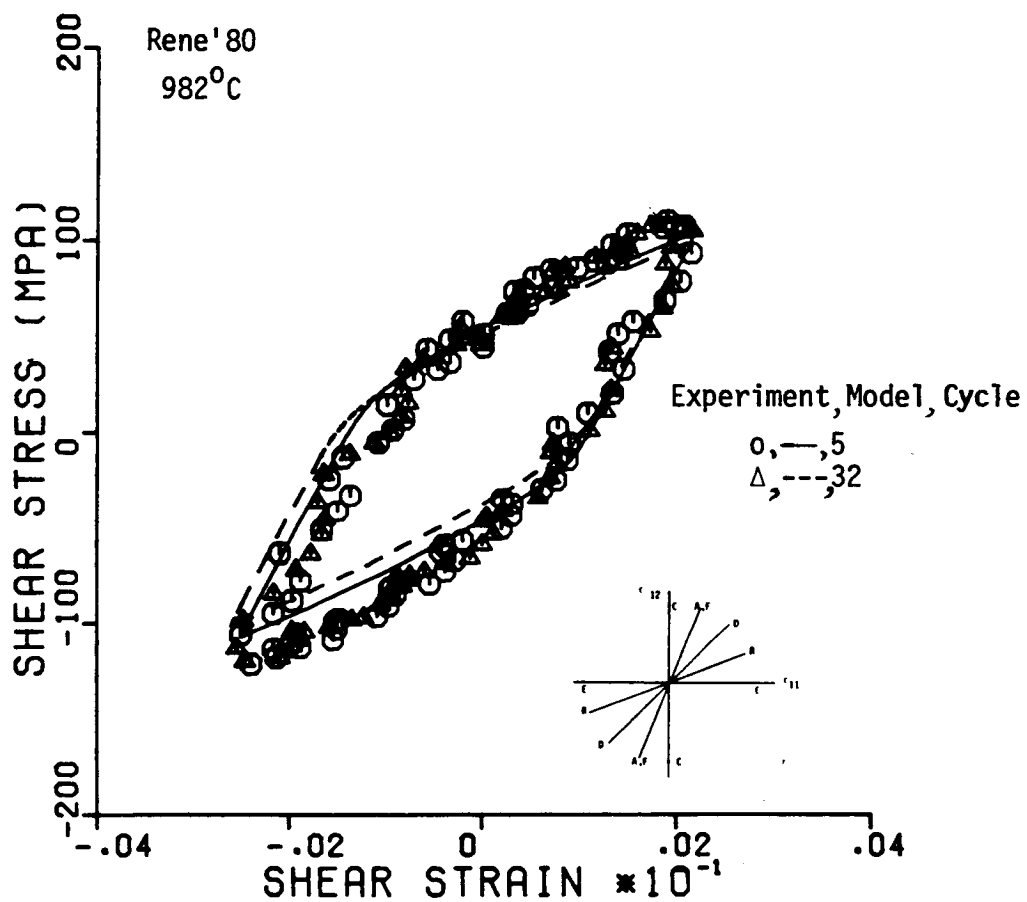


Figure 11 Comparison of Rene'80 Response Before and After Non proportional loading (Paths A,F)

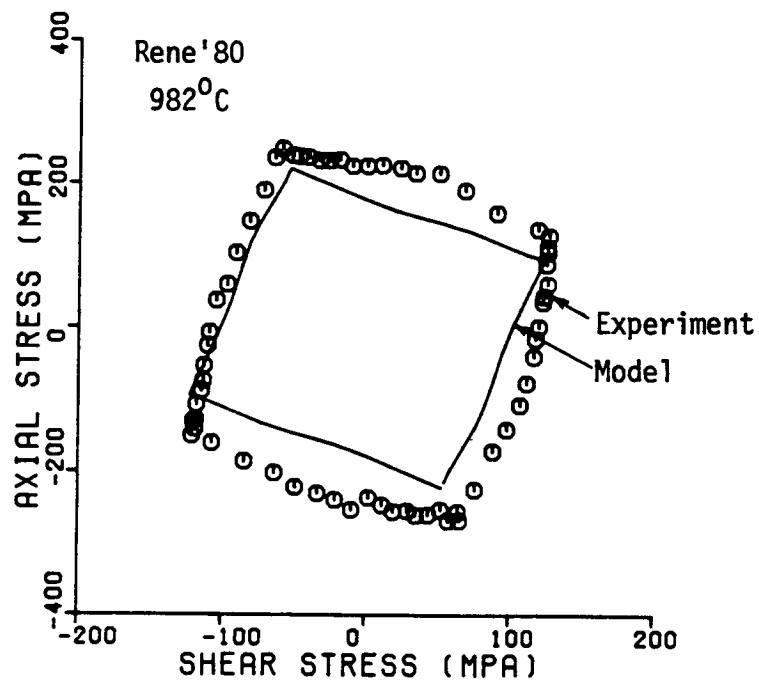


Figure 12. Rene'80 Response to 90° Out of Phase Tension/Torsion Cyclic Loading

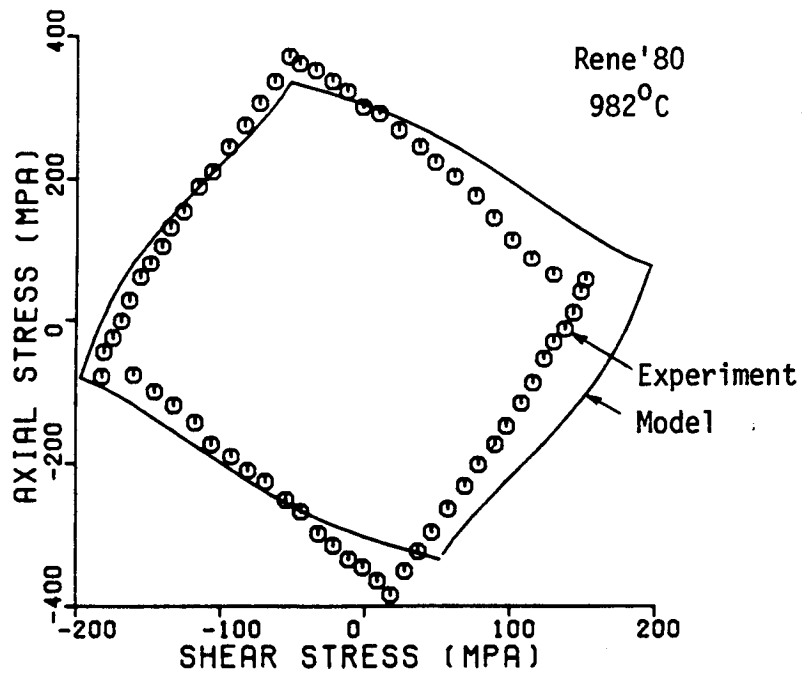


Figure 13. Rene'80 Response to 90° Out of Phase Tension/Torsion Cyclic Loading

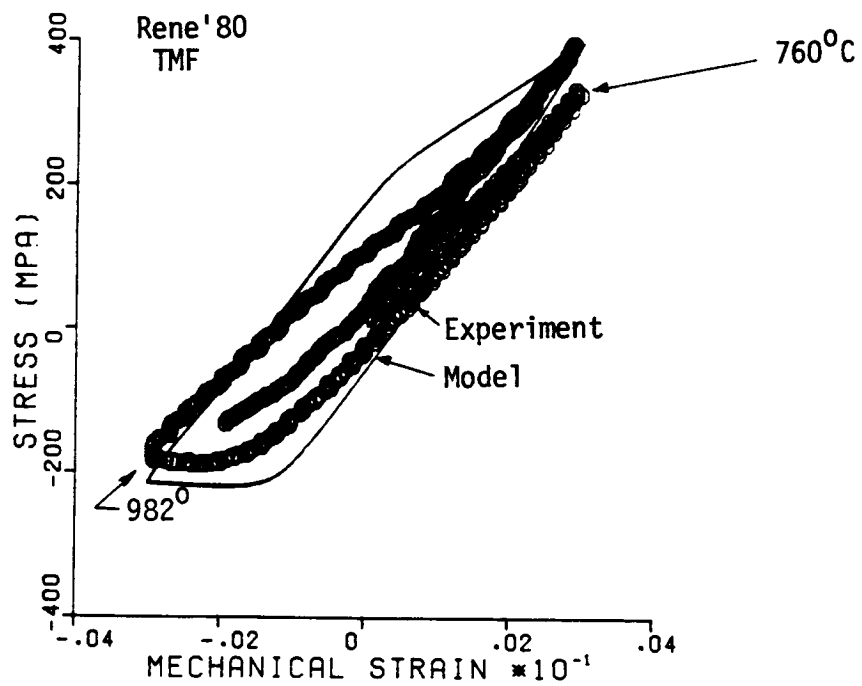


Figure 14. Rene'80 TMF Response (760°C-982°C Out of Phase)

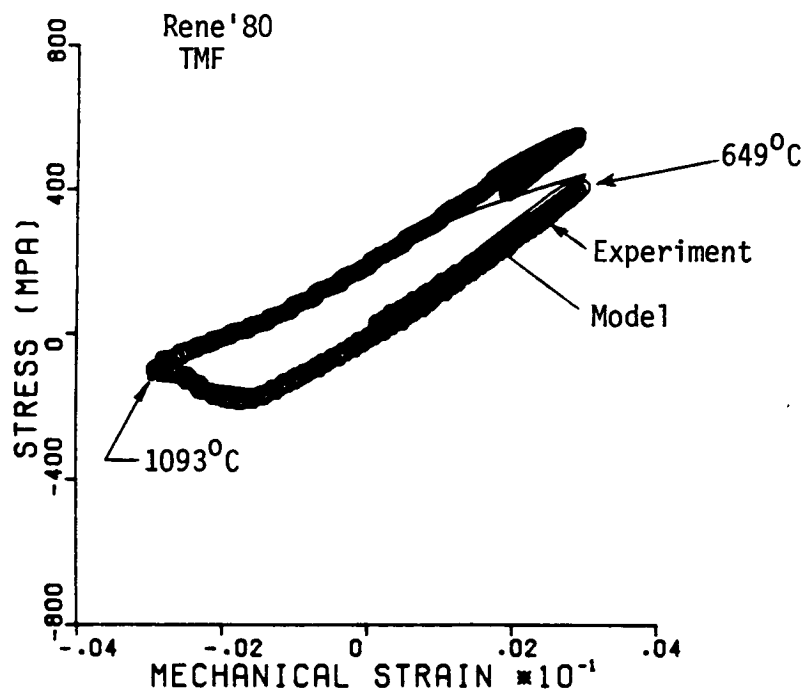


Figure 15. Rene'80 TMF Response (649°C-1093°C Out of Phase)